سكشن٤ اتصالات

$\boxed{3}$ Duality

if $g(t) \rightleftharpoons G(f)$

then $G(t) \rightleftharpoons g(-f)$

$\underline{\underline{\text{Ex}}}$ Find F.T of $A\tau \operatorname{sinc}(t\tau)$

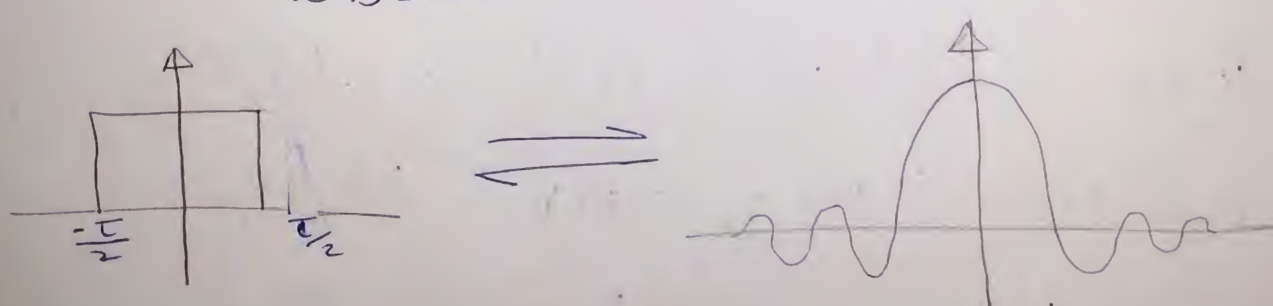
using Duality

$$A \operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \operatorname{sinc}(f\tau)$$

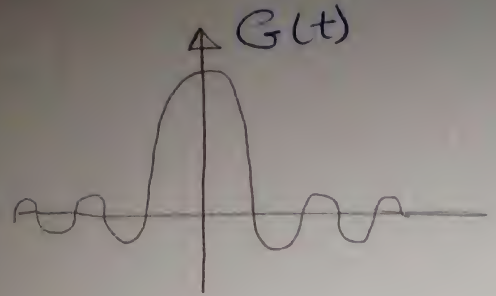
$$A\tau \operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A \operatorname{rect}\left(\frac{-f}{\tau}\right)$$

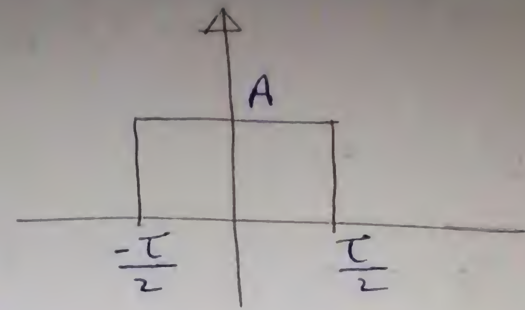
$-f=0,\ f=0$ * فى حالة ال rect فقط ممكن تشيل (-)

كلاهما واحد



Limited ~~freq~~ in time $\rightleftharpoons$ unlimited freq





* using Duality

1) $A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)$

2) $A\tau\,\text{sinc}(\tau t) \rightleftharpoons A\,\text{rect}\left(\frac{f}{\tau}\right)$

$A\sin(2wt) \rightleftharpoons \frac{A}{2w}\,\text{rect}\left(\frac{f}{2w}\right)$

$\tau = 2w$

$\boxed{\text{EX}}$ Find F.T of sine (mt)

using Duality

$A\,\text{rect}(t/\tau) \rightleftharpoons A\tau\,\text{sinc}(f\tau)$

$A\tau\,\text{sinc}(t\tau) \rightleftharpoons A\,\text{rect}(f/\tau)$

$\text{sinc}(mt) \rightleftharpoons \frac{1}{m}\,\text{rect}\left(\frac{f}{m}\right)$

$\tau = m$, $A\tau = 1$, $A = \frac{1}{m}$

$\boxed{2}$ sec 4

$\boxed{4}$ Time shift Property

If $g(t) \rightleftharpoons G(f)$

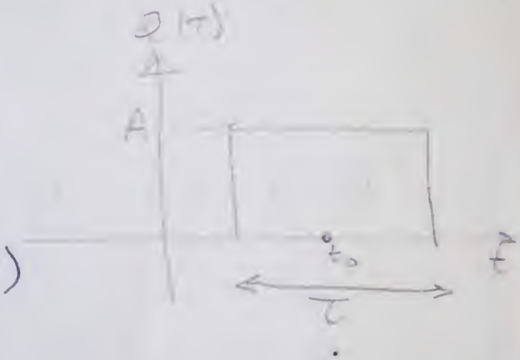
then $g(t \pm t_0) \rightleftharpoons G(f) \cdot e^{\pm J 2\pi f t_0}$

نفس الإشارة

Ex: Find F.T for $g(t) = \underset{\text{amp}}{A}\, rect\,(t - t_0)$

sol

using time shift



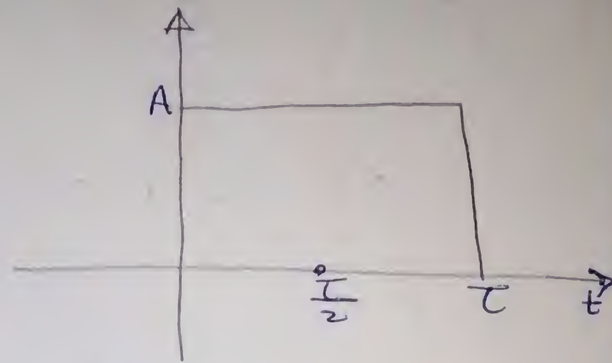
$$A\, rect\left(\frac{t}{\tau}\right) \rightleftharpoons \underbrace{A\tau \sin(f\tau)}_{G(f)}$$

$$A\, rect\left(\frac{t - t_0}{\tau}\right) \rightleftharpoons A\tau\, sinc(f\tau) \cdot e^{-J 2\pi f t_0}$$

Ex find F.T for $g(t) = A\,\mathrm{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right)$

Sol

using time shift

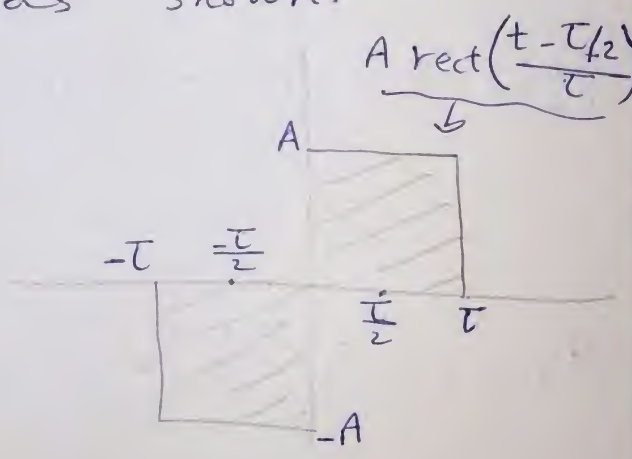


$A\,\mathrm{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right) \rightleftharpoons$

$$A\,\mathrm{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right) \rightleftharpoons A\tau\,\mathrm{sinc}(f\tau)\cdot e^{-J2\pi f\frac{\tau}{2}}$$

$$A\tau\,\mathrm{sinc}(f\tau)\cdot e^{-J\pi f\tau}$$

Ex find F.T of $g(t)$ as shown.



$g(t) = A\,\mathrm{rect}\left(\frac{t-\tau/2}{\tau}\right)$

$-A\,\mathrm{rect}\left(\frac{t+\tau/2}{\tau}\right)$

~~-A rect~~

using linearity & time shift

$$A\,rect\left(\frac{t-\tau/2}{\tau}\right) \rightleftharpoons A\tau\, sinc(f\tau)\cdot e^{-J2\pi f \frac{\tau}{2}}$$

$$A\,rect\left(\frac{t+\tau/2}{\tau}\right) \rightleftharpoons A\tau\, sinc(f\tau)\cdot e^{J2\pi f \frac{\tau}{2}}$$

$$G(f) = A\tau\ sinc(f\tau)\left[e^{-J\pi f\tau} - e^{+J\pi f\tau}\right]$$

$$* \frac{-2J}{-2J}$$

$$\therefore G(f) = (-2J)\, A\tau\, sinc(f\tau)\cdot \sin(\pi f\tau)$$

$\boxed{5}$ Frequency-shift property

if $g(t) \rightleftharpoons G(f)$

then $g(t)\cdot e^{\pm J2\pi f t} \rightleftharpoons G(f \mp f_0)$

$\underline{\underline{\text{Ex}}}$ find F.T of $A\,\text{rect}\left(\frac{t}{\tau}\right) \cdot e^{-j2\pi f_c t}$

<u>Sol</u>

using freq. shift

$$A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \;\text{sinc}(f\tau)$$

$$G(f) = A\tau \;\text{sinc}(f\tau)$$

$$G(f+f_o) = A\tau \;\text{sinc}\left(\tau(f+f_o)\right)$$

$\underline{\underline{\text{Ex}}}$ find F.T of

$\boxed{\text{هام}}$

$$x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right) \cdot \cos(2\pi f_c t)$$

$$x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right]$$

$$= \frac{1}{2}\left[A\,\text{rect}\left(\frac{t}{\tau}\right) \cdot e^{j2\pi f_c t} + A\,\text{rect}\left(\frac{t}{\tau}\right) e^{-j2\pi f_c t}\right]$$

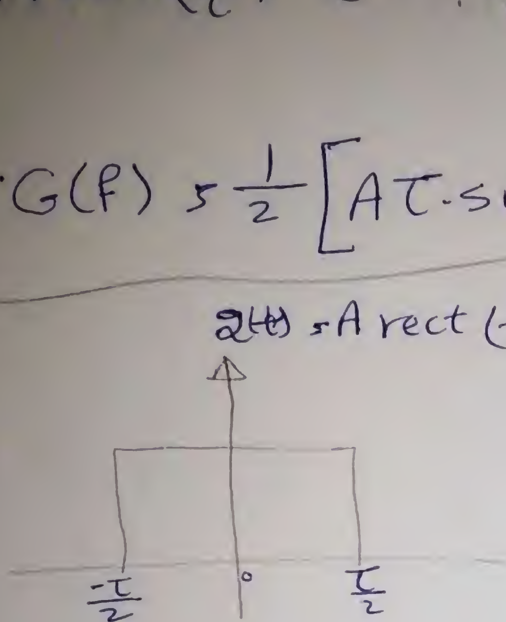
using superposition & Linearity

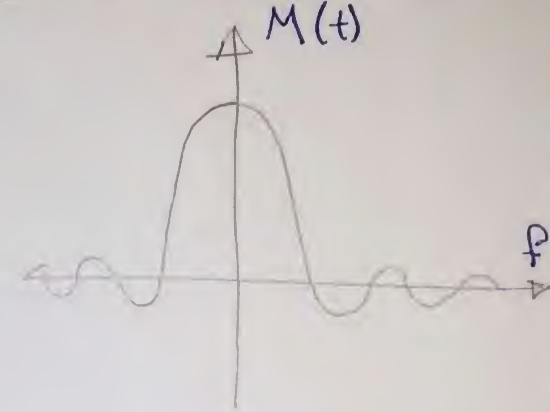
$$A\,\text{rect}\left(\frac{t}{\tau}\right) \cdot e^{j2\pi f_c t} \rightleftharpoons A\tau \;\text{sinc}\left((f-f_c)\tau\right)$$

$$A\,\mathrm{rect}\left(\frac{t}{\tau}\right)\cdot e^{-j2\pi f_c t} \rightleftharpoons A\tau\cdot \mathrm{sinc}\left((f+f_c)\tau\right)$$

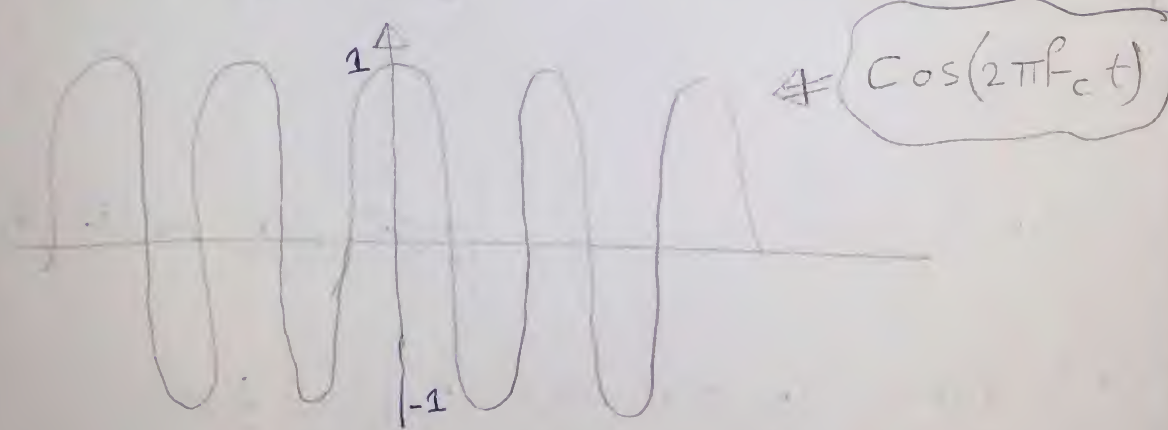
$$\therefore G(f) = \frac{1}{2}\left[A\tau\cdot \mathrm{sinc}\left((f-f_c)\tau\right) + A\tau\, \mathrm{sinc}\left((f+f_c)\tau\right)\right]$$

$g(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$

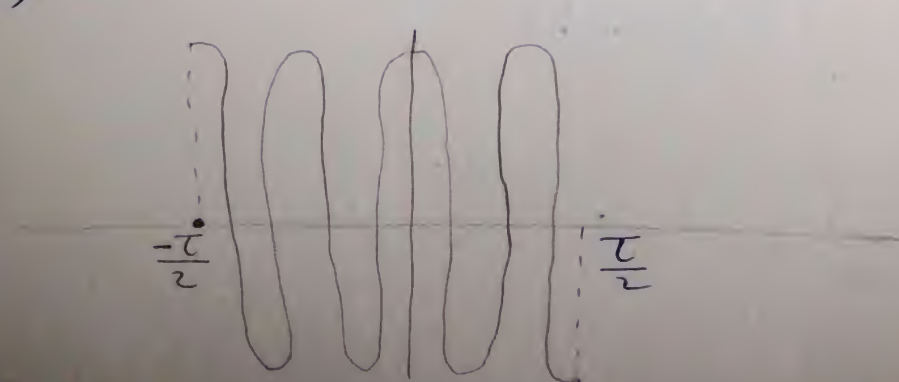




$g(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)\cdot \cos(2\pi f_c t)$

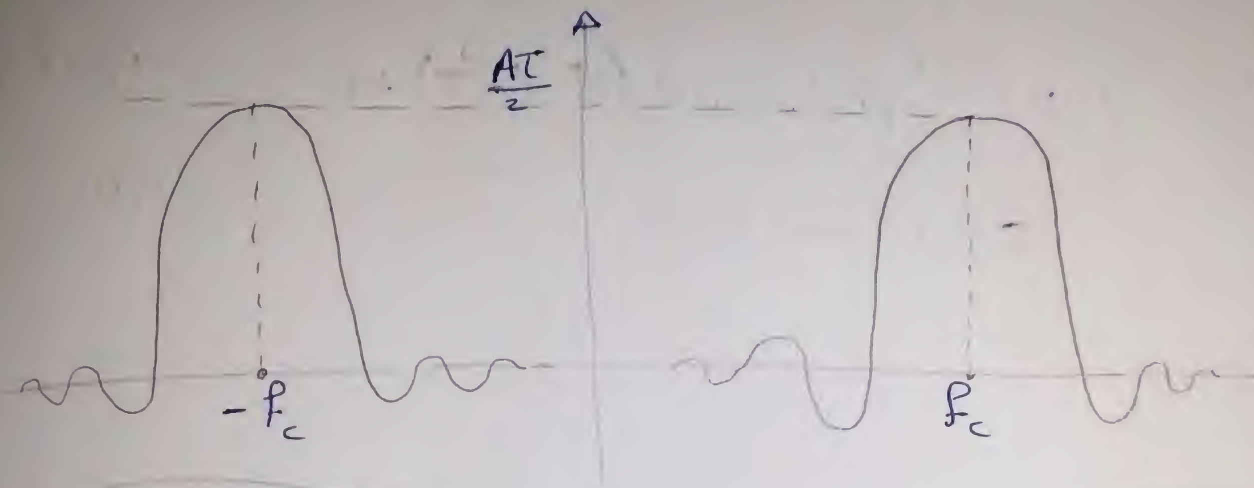


$g(t)$



$\boxed{7}$ sec 4

$$G(f) = \frac{1}{2}\left[A\tau \operatorname{sinc}((f-f_c)\tau) + A\tau \operatorname{sinc}((f+f_c)\tau)\right]$$



* Modulation Theory:-

$$m(t)\cdot \cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[M(f+f_c) + M(f-f_c)\right]$$

$$m(t)\cdot \frac{e^{J\theta} - e^{-J\theta}}{2\tau} \rightleftharpoons \frac{1}{2\tau}\left[M(f-f_c) - M(f+f_c)\right]$$

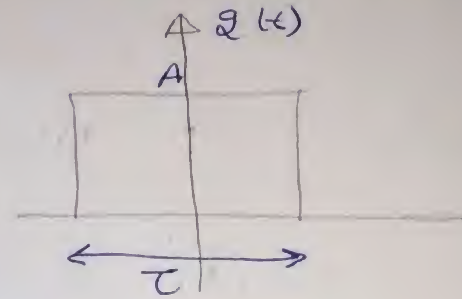
[6] Area under the curve $g(t)$

$$\text{Area} = \int_{-\infty}^{\infty} g(t)\cdot dt$$

$$G(f) = \int_{-\infty}^{\infty} g(t)\cdot e^{-J2\pi f t}\cdot dt$$

$$\therefore \text{Area} = \overset{f=0}{G(0)}$$

<u>Ex</u> find Area of $x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)$



Sol

$$\text{Area} = A * \tau$$

$$\text{Area} = A\tau$$

another solution

$$G(f) = A\tau\,\text{sinc}(f\tau)$$

$$\text{Area} = G(0) = A\tau\,\underbrace{\text{sinc}(0)}_{1} = A\tau$$

$\boxed{7}$ Area under Curve $G(f)$

$$\text{Area} = \int_{-\infty}^{\infty} G(f)\,.\,df$$

I.f.t

$$x(t) = \int_{-\infty}^{\infty} G(f)\,.\,e^{+J2\pi f.t}\,.\,df$$

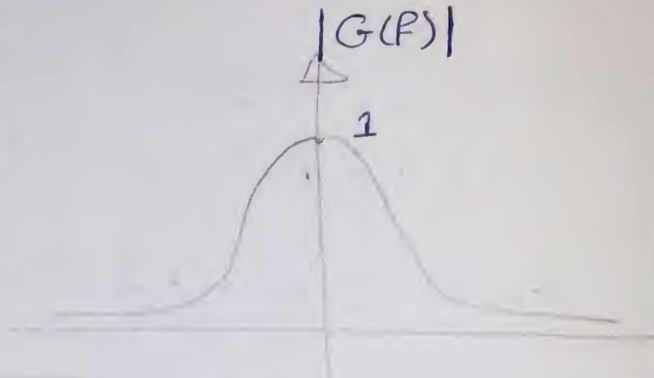
$\Rightarrow t=0$

$\therefore$ Area $= g(0)$

Ex Find Area under Curve

$$G(f) = \frac{1}{J 2\pi f}$$

$$|G(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$



$g(t)$:

$g(t) = e^{-t} \cdot u(t)$

$\therefore$ Area $= g(0)$

Area $= 1 \cdot \frac{1}{2}$

$= \frac{1}{2}$

Report 2

$\boxed{}$ Find F.T for

$\boxed{1}$ $g(t) = 3\,\mathrm{sgn}(t-3)$

$\boxed{2}$ $g(t) = \mathrm{rect}\left(\frac{t+1.5}{7}\right)$

$\boxed{3}$ $g(t) = 3 \cdot e^{-|t-2|}$

$\boxed{10}$ sec 4